

AP Calculus AB
Slope of Tangent Lines – Limit Definition

Name Key

$$m_{\text{sec}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$m_{\text{tan}} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = x^2 + 1$ at $x = 2$
 $f(2) = 5$

$$\lim_{x \rightarrow 2} \frac{x^2 + 1 - 5}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2}$$

$$f'(2) = 4$$

equation:

$$y - 5 = 4(x - 2)$$

2. $f(x) = x^3 - 2$ at $x = 1$
 $f(1) = -1$

$$\lim_{x \rightarrow 1} \frac{x^3 - 2 + 1}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{x-1}$$

$$f'(1) = 3$$

equation:

$$y + 1 = 3(x - 1)$$

3. $f(x) = 3x^2 - 2x + 5$ at $x = 4$
 $f(4) = 45$

$$\lim_{x \rightarrow 4} \frac{3x^2 - 2x + 5 - 45}{x - 4}$$

$$\lim_{x \rightarrow 4} \frac{3x^2 - 2x - 40}{x - 4}$$

$$\lim_{x \rightarrow 4} \frac{(3x+10)(x-4)}{x-4}$$

$$f'(4) = 22$$

equation:

$$y - 45 = 22(x - 4)$$

$$4. f(x) = \frac{3}{x} \quad \text{at } x=3$$

$$f(3) = 1$$

$$\lim_{x \rightarrow 3} \frac{\left(\frac{3}{x} - 1\right) x}{(x-3) x}$$

$$\lim_{x \rightarrow 3} \frac{3-x}{(x-3) x}$$

$$\lim_{x \rightarrow 3} \frac{-(-3+x)}{(x-3) x}$$

$$f'(3) = -\frac{1}{3}$$

Equation:

$$y-1 = -\frac{1}{3}(x-3)$$

$$5. f(x) = \frac{x}{x-2} \quad \text{at } x=3$$

$$f(3) = 3$$

$$\lim_{x \rightarrow 3} \frac{\left(\frac{x}{x-2} - 3\right) (x-2)}{(x-3) (x-2)}$$

$$\lim_{x \rightarrow 3} \frac{x-3(x-2)}{(x-3)(x-2)}$$

$$\lim_{x \rightarrow 3} \frac{x-3x+6}{(x-3)(x-2)}$$

$$\lim_{x \rightarrow 3} \frac{-2x+6}{(x-3)(x-2)}$$

$$\lim_{x \rightarrow 3} \frac{-2(x-3)}{(x-3)(x-2)}$$

Equation:

$$y-3 = -2(x-3)$$

$$f'(3) = -2$$